Solution to April's

ALMA COLLEGE MATH CHALLENGE

Let A, B, and C be any three distinct points in the plane. **PROVE** that it is ***always possible*** to construct a square whose sides contain these three points?

Solution

The answer is **YES**. If the points are collinear, just draw a line segment containing the points and make it the side of a square.

Now, assume the points are non-collinear. By rotating the entire collection of three points, we may assume the two points *farthest apart* are joined by a horizontal line segment.

Now, rotate by 180°, if needed, so that the third point is above the horiontal line segment, and label the points A, B, and C, as in Figure 1. Note that C must be inside the square with side $\overline{AB}$ since A and B are the two points farthest apart.

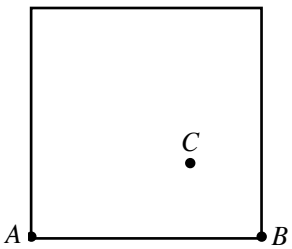

Figure 1

Now slide the square down until C is on the top side. (Figure 2) By construction, A and B are also on the sides of the square. That's it.

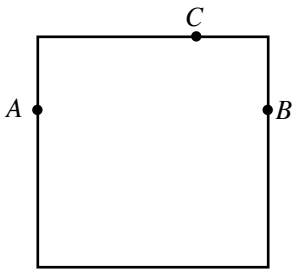

Figure 2

There are many other construction methods that will also work. ■

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