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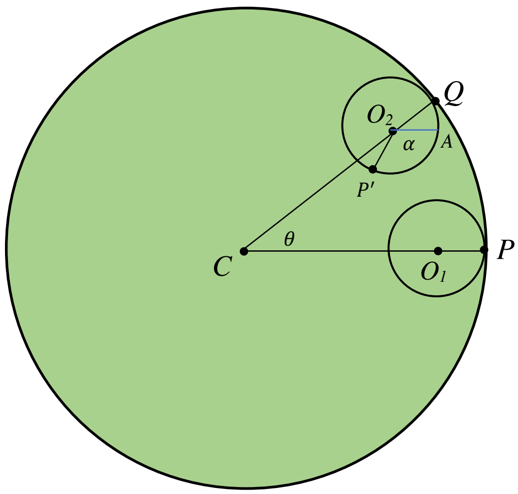
Solution to March's

ALMA COLLEGE MATH CHALLENGE

A circle of radius 1 is inside a circle of radius 5. The smaller circle is rotated clockwise on the larger circle without slipping until it arrives back at its starting point ***P***. How many revolutions (turns) will the small circle make? Be careful. *The answer is not 5*.



Solution by ***Morrison Ma****, Western Michigan Christian High School*

Mark the center of the larger circle with C, and mark the center of the smaller cicle with O_1. Now, rotate $\odot O_1$, this is the notation we'll use to identify the circle with center O_1, clockwise inside of $\odot C$ stopping at $\odot O_2$. Let Q be the point at which $\odot O_2$ touches $\odot C$.

Draw $AO_2 \parallel CP$, and let AO_2 intersect $\odot O_2$ at A. Let $\angle AO_2P' = \alpha$, and let $\angle C = \theta$. Then we can say $\angle AO_2Q = \angle C = \theta$.

We know $\angle AO_2P' = \alpha$, and we also know that the length the smaller (unit) circle has been rolled is $\left|\widehat{QP'}\right| = \theta + \alpha = \left[\widehat{QP}\right]$. In $\odot C$, $\left|\widehat{QP}\right| = 5\theta$. So, $\theta + \alpha = 5\theta$, which implies that $\alpha = 4\theta$. That means, when the smaller circle makes one complete rotation, $\alpha = 2\pi$ and $\theta = \frac{\pi}{2}$. So, one rotation of the smaller circle will cover $\frac{1}{4}$ of the circumference of the larger circle, which means the smaller circle will have made 4 revolutions to make it back to P. ■

Also solved by:

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