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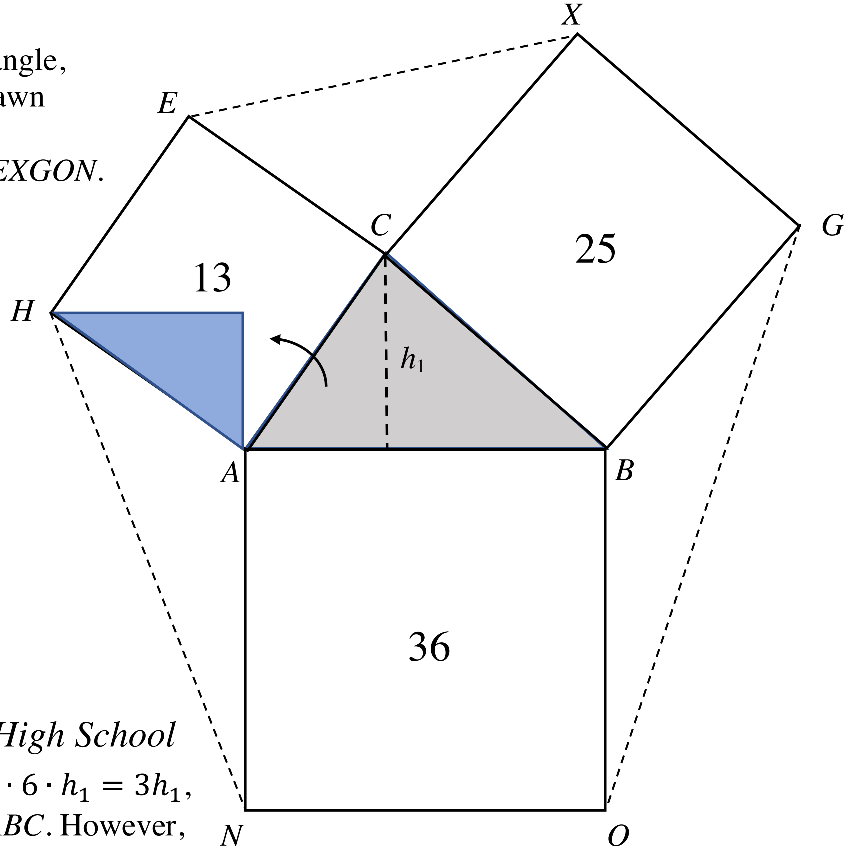
Solution to February's

ALMA COLLEGE MATH CHALLENGE

Triangle ABC, which is not a right triangle, has squares of areas 13, 25, and 36 drawn on its sides.
Find the *exact area* of the hexagon $HEXGON$.



Solution by ***Auden Chase****, Holt High School*

The area of triangle ABC is given by $\frac{1}{2} \cdot 6 \cdot h_1 = 3h_1$, where h_1 is the height of the triangle ABC. However, if we take the inner right triangle formed by A, C, and the altitude of length h_1, we can spin it about vertex A such that side AC meets side AH.

As you can see, the triangles must fit because $ACEH$ is a square, and angles HAN and CAB are supplementary: $\angle HAN + \angle CAB = 360 - 90 - 90 = 180$.

Therefore, since triangle ABC and HAN have the same base and height h_1, they must have the same area. A similar procedure can be applied to the two other triangles. Therefore, the area of the hexagon $HEXGON$ is: $A = 36 + 25 + 13 + 12h_1$, where $h_1 = sinA \cdot \sqrt{13}$.

The measure of angle A can be found with the law of cosines.
Beginning with $25 = 36 + 13 - 2 \cdot 6 \cdot \sqrt{13} \cdot cosA$, we solve and get $cosA = \frac{2}{\sqrt{13}}$.

Therefore, since $sin^2A + cos^2A = 1$, we have $sinA = \frac{3}{\sqrt{13}}$.

So, $A = 36 + 25 + 13 + 12 \cdot \frac{3}{\sqrt{13}} \cdot \sqrt{13} = 36 + 25 + 13 + 36 = 110$. ■

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