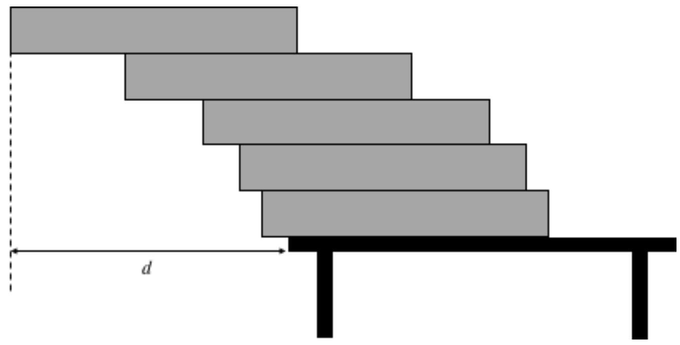
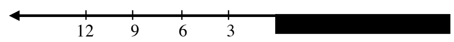
Solution to December's
ALMA COLLEGE MATH CHALLENGE

After unwrapping the presents on Christmas morning, you find five identical empty boxes, each 12 inches long. Your parents want you to help with the clean-up, but you decide to do something more interesting with the five boxes. Suppose you carefully cantilever them so that the boxes extend out over the edge of a table. Using no glue, tape, or any means of joining the boxes together, what is the MAXIMUM DISTANCE ***d*** you can extend the five boxes beyond the edge of the table? Assume that the center of mass of each box is at the geometric center of the box. Please justify your answer, and please express your answer as an exact fraction.



*Solution by **Raayan Brar**, Ann Arbor Huron High School*

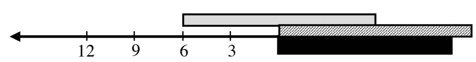
Let's say there is a number line in inches at the left of the table.



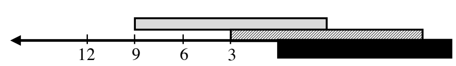
To make sure the boxes don't fall over, the center of mass of the stack of boxes must be on the table. To achieve the maximum overhang, the center of mass must lie at 0 on the number line, which is at the left edge of the table.

Starting with one box, the overhang would be 6 inches because the center of mass is at the middle of the 12 inch box.

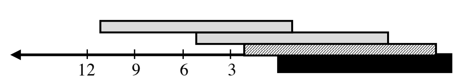
When we add a second box (beneath the first), we let the left edge of the box touch the edge of the table, which is at 0.



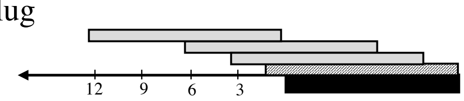
To calculate ***c***, the center of mass of these two boxes, we use the formula $c = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$, where x_1 and x_2 are where the two objects' centers of mass lie on the number line, and where m_1 and m_2 are the masses of the two objects. We'll assume that each box has a mass of 1. So, we plug in the values, $x_1 = 0, x_2 = -6$, and $m_1 = m_2 = 1$, and get $c = \frac{(0 \cdot 1) + (-6 \cdot 1)}{1+1} = -3$. This means that the center of mass of the two boxes is at -3 on the number line. Now, we can safely move the boxes 3 inches to the left.



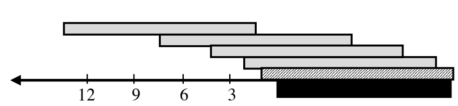
Now let's add a third box underneath the top two so that its left side is at 0 on the number line. To calculuate the center of mass of this stack, we can think of the two previously stacked blocks as one object with a mass of 2. Substituting $x_1 = 0, x_2 = -6, m_1 = 2$, and $m_2 = 1$, we get $c = \frac{(0 \cdot 2) + (-6 \cdot 1)}{2+1} = -2$, which means we can shift the stack of three boxes 2 inches to the left.



Adding a fourth box under the previous stack of three, we can plug in the values $x_1 = 0, x_2 = -6,\ m_1 = 3$, and $m_2 = 1$, and get $c = \frac{(0 \cdot 3) + (-6 \cdot 1)}{3+1} = -\frac{3}{2}$, which means we can shift the stack of four boxes $\frac{3}{2} = 1.5$ inches to the left.



Finally, adding a fifth box under the four previous ones, we get $c = \frac{(0 \cdot 4) + (-6 \cdot 1)}{4+1} = -\frac{6}{5}$, which allows us to shift the stack of five boxes $\frac{6}{5} = 1.2$ inches to the left.



Therefore, the maximum distance the five boxes can extend over the edge of the table is $6 + 3 + 2 + \frac{3}{2} + \frac{6}{5} = \frac{137}{10} = 13.7$ inches. ■

Also solved by: **Prathik Guduri**, **Jerry Li**, **Joey Oh**, **Marisa Redding**, **Jiyoon Shin**, **Mukesh Sivakumaran**, **Brian Wu**, **Dimash Umirbayev** (Ann Arbor Huron H.S.); **Parker Bundy**, **Grant Carr**, **Madeline Carr**, **Nolan Day**, **Alec Kussro** (Fenton H.S.); **Suchir Gupta** (Forest Hills Central H.S.); and **Christian Martinez** (Berrien Springs H.S.).

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