Solution to February’s 
ALMA COLLEGE MATH CHALLENGE

Our country has had 44 presidents. Write each president’s last name on a card and place all 44 cards in a box. Note that several names (e.g., Cleveland) occur twice. Now, randomly select 2 cards from the box.

What’s the probability that the 2 names have the same number of letters?

Solution by Kenneth Steinhilber, Midland High School

There are 44 presidents, and we find there are 5 four-letter last names, 7 five-letter last names, 9 six-letter last names, 8 seven-letter last names, 8 eight-letter last names, 5 nine-letter last names, and 2 ten-letter last names.

To calculate the probability, we must multiply the probability of getting an \( n \)-letter last name on the first card followed by the probability of getting an \( n \)-letter last name again (without replacement) on the second card. We must do this for each value of \( n \) (4 through 10), and then add these products together.

- Probability of selecting 2 four-letter last names: \( \frac{5}{44} \cdot \frac{4}{43} = \frac{20}{1892} \)
- Probability of selecting 2 five-letter last names: \( \frac{7}{44} \cdot \frac{6}{43} = \frac{42}{1892} \)
- Probability of selecting 2 six-letter last names: \( \frac{9}{44} \cdot \frac{8}{43} = \frac{72}{1892} \)
- Probability of selecting 2 seven-letter last names: \( \frac{8}{44} \cdot \frac{7}{43} = \frac{56}{1892} \)
- Probability of selecting 2 eight-letter last names: \( \frac{5}{44} \cdot \frac{8}{43} = \frac{40}{1892} \)
- Probability of selecting 2 nine-letter last names: \( \frac{3}{44} \cdot \frac{1}{43} = \frac{3}{1892} \)
- Probability of selecting 2 ten-letter last names: \( \frac{2}{44} \cdot \frac{1}{43} = \frac{2}{1892} \)

Finally, we sum these probabilities to obtain the total probability of selecting 2 names with the same number of letters.

\[
\frac{20}{1892} + \frac{42}{1892} + \frac{72}{1892} + \frac{56}{1892} + \frac{56}{1892} + \frac{40}{1892} + \frac{3}{1892} + \frac{2}{1892} = \frac{268}{1892} = \frac{67}{473} \cdot  
\]

Also solved by:

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